- Finish pf of Thm6 + continuity principle from Lecture 16 notes.

We can now show that pseudoconvexity is <u>local property</u> near $\partial\Omega$.

<u>Thm1</u>. Let $\Omega \subseteq \mathbb{C}^n$. If $\forall \zeta \in \partial\Omega$ $\exists$ open nbhd $U_\zeta \subseteq \mathbb{C}^n$ s.t. $\Omega \cap U_\zeta$ is ψcvx, then Ω is ψcvx.

<u>Pf</u>. For each $\zeta \in \partial\Omega$ $\exists$ $U'_\zeta \subseteq U_\zeta$ s.t. $\delta(z, \mathbb{C}^n \setminus \Omega) = \delta(z, \mathbb{C}^n \setminus (\Omega \cap U_\zeta))$, for $z \in \Omega \cap U'_\zeta$.

By assumption, $-\log \delta(z, \mathbb{C}^n \setminus (\Omega \cap U_\zeta))$ is PSH in $\Omega \cap U_\zeta$ $\Rightarrow$

$u(z) := -\log \delta(z, \mathbb{C}^n \setminus \Omega)$ is PSH in $\Omega \cap U'_\zeta$. $\Rightarrow$ $\exists$ closed $F \subseteq \Omega$ s.t. u is PSH in $\Omega \setminus F$. Let φ be a convex fcn of $|z|$ s.t. $\varphi(|z|) > u(z)$ on F. (Consider $M(r) = \sup_{z \in F, |z| \le r} u(z)$. Then $M(r) \nearrow$. Now let $\varphi(r)$ be convex majorant.) Since $\varphi \in PSH(\mathbb{C}^n)$ (by convexity), $\varphi > u$ on open nbhd of F, the fcn $v = \max(u, \varphi)$ is $PSH \cap C$ in Ω. Clearly, $\Omega_c := \{z \in \Omega : v(z) < c\}$ is precompact in Ω (i.e. satisfies (ii) in Thm6) $\Rightarrow$ Ω is ψcvx. $\blacksquare$